UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
FINAL ORDINARIO (*Final Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: Temas Selectos de Optimización

LEARNING UNIT: *Selected Topics on Optimization* (in English)
SEMESTER: August – December 2023 (Fall)
ACADEMY: *Statistics and Operations Research* (Estadística e Investigación de Operaciones).
INSTRUCTOR: Dr. Roger Z. Ríos Mercado (ID 090969)

DIRECTIONS.- Answer the following questions and/or exercises in the answer sheet. Do not write in this sheet

SECTION 1: QUESTIONS (50 POINTS)

Answer and justify your answer.

1. [*UT2: Heuristics*; 6 pts] Explain when it is appropriate to use heuristics for solving a combinatorial optimization problem.
2. [*UT1: Combinatorial optimization*; 6 pts] When do we say a combinatorial optimization problem is "hard" to solve? Elaborate.
3. [*UT1: Combinatorial optimization*; 6 pts] Are there "easy" combinatorial optimization problems? Justify your answer.
4. [*UT2: Constructive heuristics;* 6 pts] Do constructive heuristics guarantee to find a feasible solution to a given combinatorial optimization problem? Justify your answer.
5. [*UT2: Constructive heuristics;* 6 pts] Explain clearly what a construction heuristic is for a combinatorial optimization problem.
6. [*UT2: Local search heuristics;* 6 pts] Explain clearly what a local search heuristic is for a combinatorial optimization problem.
7. [*UT2: Local search heuristics for the TSP;* 7 pts] Describe in detail the 2-OPT heuristic for the Traveling Salesman Problem. You may additionally illustrate your idea with an example or drawing.
8. [*UT2: Heuristics;* 7 pts] Explain in detail how would you compare two different heuristics for a given combinatorial optimization problem, that is, how would you determine which one is better.

SECTION 2: PROBLEMS (50 POINTS)

9. The Minimum Weight k-Tree Problem (MWkTP) is defined as follows. Given a graph $G = (V, E)$, where $V = \{1, 2, \ldots, n\}$ is a set of nodes or vertices and E is a set of edges (links between nodes), a weight w_{ij} for each edge (i, j) in E, and an integer positive number k, we must find a k-tree of minimum weight. A *tree* T is defined as an acyclic and connected subgraph of G, that is, T is a subcollection of edges (and corresponding nodes incident to the edges in T) of G with the following two properties: (i) the edges and nodes in T form a connected subgraph, that is, every pair of nodes in T are connected by a path of edges entirely in T, and (ii) the edges and nodes in T form an acyclic subgraph, that is, there are no cycles in T. A k-tree is a tree with k edges (and consequently $k + 1$ nodes). Let T be a tree, then the weight of tree T is computed by adding the weight of each edge in T, that is,

$$w(T) = \sum_{(i,j) \in T} w_{ij}$$

Figure 1 shows an example of a graph $G = (V, E)$ with $V = \{1, 2, \ldots, 12\}$ and $E = \{(1,2), (1,4), (2,3), \ldots, (10,11), (11,12)\}$, that is, 12 nodes and 21 edges. Figures 2-5 illustrate certain cases when a subgraph is a tree or not.

The following questions refer to the MWkTP instance depicted in Figure 1, assuming $k = 4$.

(a) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(1)} = \{ (6,7), (6,8), (6,9), (7,8) \}$ a feasible solution? Justify your answer.

(b) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(2)} = \{ (3,4), (3,5), (3,6), (6,8) \}$ a feasible solution? Justify your answer.

(c) [*UT1: Combinatorial optimization;* 8 pts] Among the following three solutions, sort them from best to worst. Justify your answer.
$X^{(3)} = \{ (5,9), (6,8), (6,9), (7,8) \}$,
$X^{(4)} = \{ (4,6), (4,7), (5,6), (6,7) \}$,
$X^{(5)} = \{ (3,4), (3,6), (4,7), (6,8) \}$.

(d) [*UT2: Constructive heuristics;* 12 pts] Starting from scratch, design a constructive heuristic for finding a feasible solution to the MWkTP in **any** given graph G with n nodes and m edges, and given value of k. Show very clearly and with precision each step of your heuristic either in pseudocode or flow chart.

(e) [*UT2: Constructive heuristics;* 5 pts] Illustrate how your heuristic works by applying it **step by step** in the example (Figure 1) to build a feasible solution to the problem. Was this solution better than solution $X^{(3)}$ from (c)?

(f) [*UT2: Local search heuristics;* 10 pts] Given a feasible solution to the MWkTP, design a local search heuristic for the problem. It suffices with describing **very clearly** how you define your move/neighborhood.

(g) [*UT2: Local search heuristics;* 5 pts] Illustrate how your local search works starting from the following feasible solution $X^{(3)} = \{ (5,9), (6,8), (6,9), (7,8) \}$. You may do just one **complete** iteration under the best found (BF) strategy, clearly defining the entire neighborhood. Did the solution improve?

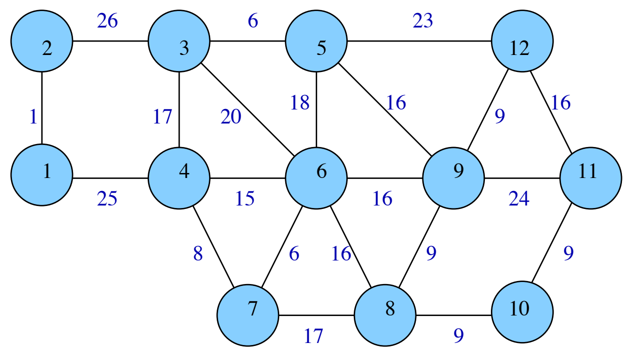

Figure 1: An instance of the MWkTP. The number inside each node indicates the node label. The number next to each edge indicates its weight.

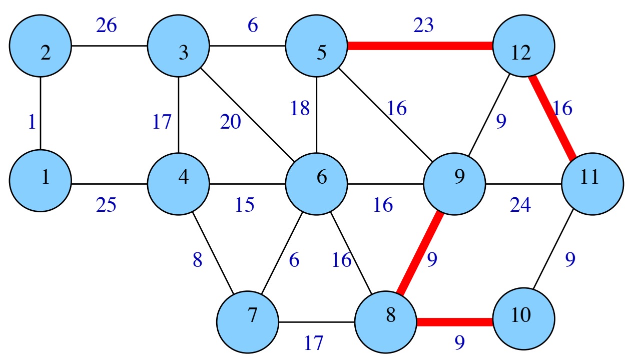

Figure 2: Subgraph shown in bold/red, T^1 = {(5,12), (11,12), (8,9), (8,10)}, is not a tree because it is disconnected.

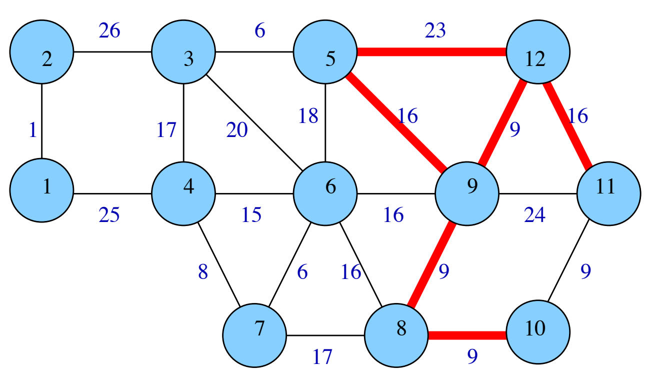

Figure 3: Subgraph shown in bold/red, T^2 = {(5,12), (5,9), (8,9), (8,10), (9,12), (11,12)}, is not a tree because it contains a cycle C = (5-9-12).

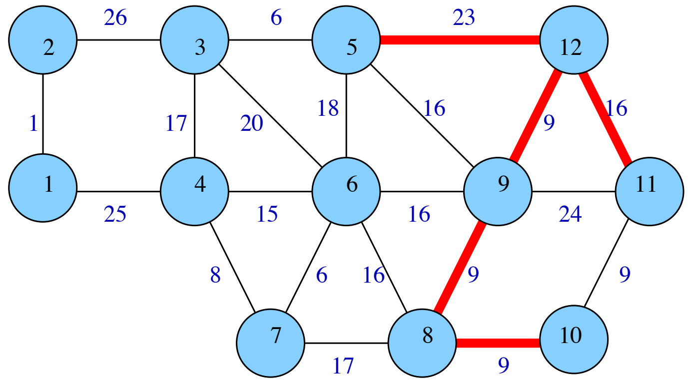

Figure 4: Subgraph shown in bold/red, $T^3 = \{(5,12), (8,9), (8,10), (9,12), (11,12)\}$, is a tree, but it is not a 4-tree because it does not have 4 edges. In fact, it is a 5-tree.

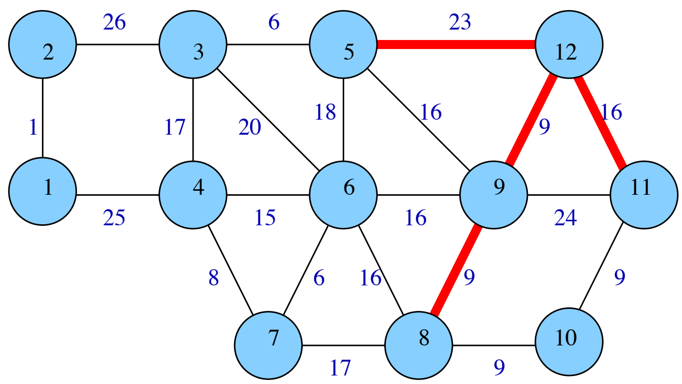

Figure 5: Subgraph shown in bold/red, $T^4 = \{(5,12), (8,9), (9,12), (11,12)$, is a 4-tree. Its weight is given by $w(\, T^4\,) = w_{5,12} + w_{8,9} + w_{9,12} + w_{11,12} = 23 + 9 + 9 + 16 = 57$.